

PROBLEM SET 1

*Assigned: January 19, 2018**Due: February 2, 2018*

Always provide explanations and show as much work as possible. Solutions to odd-numbered exercises are available at [http://www.algorist.com/algowiki/index.php/The_Algorithms_Design_Manual_\(Second_Edition\)](http://www.algorist.com/algowiki/index.php/The_Algorithms_Design_Manual_(Second_Edition)). If you are stuck on an even problem, try to find a similar odd-numbered exercise.

1. Exercise 1-2 from Skiena.
2. 1-5.
3. ~~The version of the knapsack problem in exercise 1-5 asks for a subset that sums exactly to a given parameter T . Notice that sometimes there is no solution for a given value of T . You are given an algorithm A that solves this problem in time $T(n)$. Consider a variant of this problem: find a subset of S that sums to the highest possible value $\leq T$. Give an algorithm that uses A as a procedure to solve this variant in time $O(T(n) \cdot \log T)$. Briefly explain (a formal proof is not necessary) its correctness and running time.~~
4. Your classmate claims that all jelly beans are the same color. They give the following proof:

Proof. (By induction.) Base case: when we have a single jelly bean it can only be one color.

Inductive hypothesis: Assume that all sets of n jelly beans are the same color. Consider a set of $n + 1$ jelly beans. Choose a subset A of size n from this set; these beans must be the same color by our inductive hypothesis.

Now consider the bean you did not pick. Swap this bean with any bean from the set A . This forms a set B of n beans, and so must be the same color by our inductive hypothesis.

Sets A and B share $n - 1$ beans, hence each set must be the same color. In other words, all $n + 1$ beans are the same color. \square

What is wrong with this proof?

5. 1-8
6. 1-12
7. 1-17
8. 1-20. Interviewers love to ask estimation questions like this. The important part here is your *process*, not the actual answer you get. Make some simplistic assumptions and ballpark it.
9. 2-2
10. 2-6
11. 2-8
12. 2-12
13. 2-18
14. 2-23
15. 2-36